**R09**

**Code No: D7501**

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**M.Tech II - Semester Examinations, March/April 2011**
**OPTIMAL CONTROL THEORY**
**(CONTROL SYSTEMS)**

**Time: 3hours** **Max. Marks: 60**

**Answer any five questions**
**All questions carry equal marks**
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1. a) State and explain the Bellman's principle of optimality.
   b) Explain the concept of applying the principle of optimality in decision making process to get optimal trajectory. [6+6]

2. a) Derive recurrence relation for time-invariant systems of dynamic programming for N-stage process.
   b) Explain the characteristics of dynamic programming solution. [6+6]

3. A first – order linear system is given $\dot{x} = -10x(t) + u(t)$ is to be controlled to minimize the performance measure

$$J = \frac{1}{2}x^2(0.04) + \int_0^{0.04} [\frac{1}{4}x^2(t) + \frac{1}{2}u^2(t)]\,dt$$

The admissible state and control values are not constrained by any boundaries. Find the optimal control law by using the Hamilton-Jacobi-Bellman equation. [12]

4. a) State and explain the fundamental theorem of the calculus of variations.
   b) Determine an extremal for the functional

$$J(x) = \int_0^2 [4x^2(t) + 2x(t)\dot{x}(t) + \dot{x}^2(t)]\,dt$$

The boundary conditions are x(0)=1, and x(2) is free. [5+7]

5. The system is given by:

$\dot{x}_1(t) = x_2(t)$

$\dot{x}_2(t) = -x_1(t) + [1 - x_1^2(t)]x_2(t) + u(t)$ is to be transferred from the origin to the surface

$[x_1(t) - 4]^2 + [x_2(t) - 5]^2 + [t - 2]^2 = 9$

with minimum fuel expenditure. The final time is free, and $|u(t)| \le 1.0$

   a) Determine the costate equations.
   b) Determine the control that minimizes the Hamiltonian.
   c) Determine the boundary conditions at $t=t_f$. [4+4+4]

6. State the Two-point boundary value problems and explain how it can be solved though variation of extremals. Also summarize the algorithm of variation of extremals. [12]

**Contd…..2**

7. State and explain the state estimation problem and with suitable block diagram derive mathematical model of the non-statistical full order estimator. Also summarize its algorithm. [12]

8. Consider the plant $\dot{x}(t) = x(t) + v$, y(t) = x(t)+w, with $E[v(t)v(\tau)] = E[w(t)w(\tau)] = \delta(t-\tau)$ and *v* and *w* independent. Suppose that at time zero, x(0) is known to be zero. Design an optimal estimator. [12]

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